

RIJKSUNIVERSITEIT GRONINGEN

DTFE

The Delaunay Tessellation Field Estimator

PROEFSCHRIFT

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Cover illustration:

DTFE reconstruction of the large scale matter distribution in the universe.

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