9

Summary and Outlook

9.1 Summary

In this thesis we have described a new tool for reconstructing a volume-covering and continuous density field from a discrete point sampling. We have shown that this tool, the *Delaunay Tessellation Field Estimator* (DTFE), is very well suited for the analysis of highly complex point distributions, such as the large scale galaxy distribution and cosmological N-body simulations.

9.1.1 The Delaunay Tessellation Field Estimator

In Chapters 2 and 3 we have extensively described the DTFE. In essence the DTFE is a stochastic-geometrical algorithm for transforming a given discrete point sampling to its corresponding continuous density field. It involves an extension of the interpolation procedure described by Bernardeau & van de Weygaert (1996), who introduced the Delaunay tessellation (Delone 1934, see also Okabe 2000 and references therein) of a point set as a natural and self-adaptive frame for multi-dimensional interpolation.

The DTFE consists of three main steps, which are illustrated in Fig. 9.1. The starting point is a given discrete point distribution. In the upper left-hand frame of Fig. 9.1 a point distribution is plotted in which at the center of the frame an object is located whose density diminishes radially outwards. In the first step of the DTFE the Delaunay tessellation of the point distribution is constructed. This concerns a volume-covering division of space into triangles (tetrahedra in three dimensions), whose vertices are formed by the point distribution (Fig. 9.1, upper right-hand frame). The Delaunay tessellation is defined such that inside the interior of the circumcircle of each Delaunay triangle no other points from the defining point distribution are present.

The Delaunay tessellation forms the heart of the DTFE. In Fig. 9.1 it is clearly visible that the tessellation automatically adapts to the both the local density and geometry of the point distribution: where the density is high, the triangles are small and vice versa. The size of the triangles is therefore a measure of the local density of the point distribution. This property of the Delaunay tessellation is exploited in step 2 of the DTFE, in which the local density is estimated at the locations of the sampling points. For this purpose the density is defined at the location of each sampling point as the inverse of the area of its surrounding Delaunay

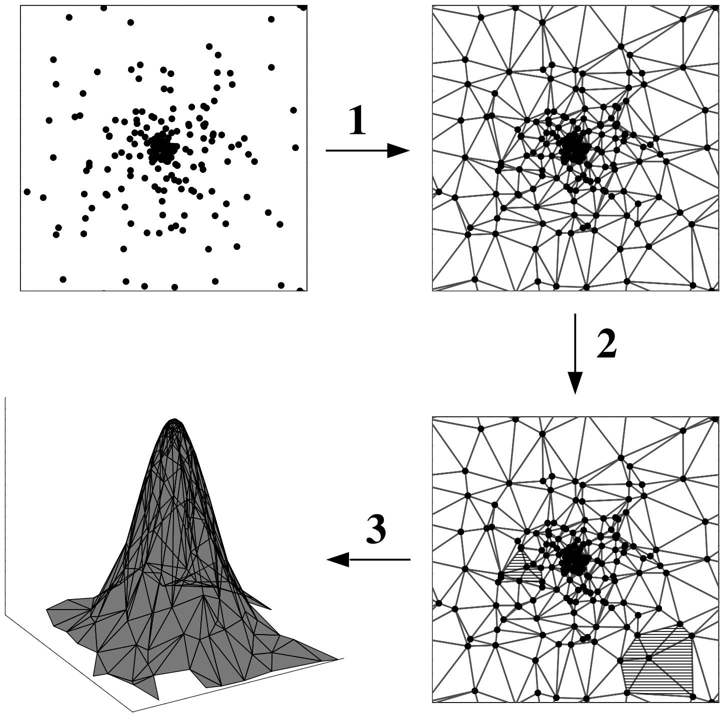

Figure 9.1 — Overview of the DTFE reconstruction procedure. Given a point distribution (top left), one has to construct its corresponding Delaunay tessellation (top right), estimate the density at the position of the sampling points by taking the inverse of the area of their corresponding contiguous Voronoi cells (bottom right) and to assume that the density varies linearly within each Delaunay triangle, resulting in a volume-covering continuous density field (bottom left).

triangles (times a normalization constant, see Fig. 9.1, lower right-hand frame).

In step 3 these density estimates are interpolated to any other point, by assuming that inside each Delaunay triangle the density field varies linearly (Fig. 9.1, lower left-hand frame).

In essence the DTFE is a first-order multi-dimensional interpolation scheme which is closely related to the more generic higher-order natural neighbor procedure (Sibson 1981, see also Watson 1992 and Sukumar 1998). This smooth and local spatial interpolation technique is the most general and robust method of interpolation available to date. However, the implementation of natural neighbour interpolation has been hampered by the fact that an efficient implementation in three dimensions is not trivial. Moreover, it is not clear how a density field reconstruction scheme based on natural neighbour interpolation would conserve mass.

The DTFE allows us to follow the same geometrical and structural adaptive properties of the higher-order natural-neighbour method, while allowing the analysis of truely large data sets and making sure that mass is conserved. The most important property of the DTFE is that it automatically adapts to both the local density and geometry of the point distribution. This results in important advantages with respect to conventional reconstruction procedures. In this thesis we have extensively tested the DTFE and compared its performance with conventional reconstruction methods.

In Chapter 3 we have shown that of all studied reconstruction procedures the DTFE attains the highest spatial resolution. In Chapters 4 and 5 we have specifically studied the performance of the DTFE with respect to three key aspects of the large scale galaxy distribution:

- hierarchical clustering;
- anisotropic collapse;
- a complex cellular geometry with extended empty regions.

We have shown that with respect to all these aspects the DTFE significantly outperforms existing reconstruction procedures.

The very high spatial resolution of the DTFE makes it also sensitive to sampling noise and/or measurement errors. In Chapter 8 we have described and quantified the statistical properties of the DTFE. We have discussed how the significance of reconstructed density fields can be determined and what the influence is of several types of errors and uncertainties on the reconstructed field.

9.1.2 An atlas of the nearby universe

In this thesis we have described a number of applications for which the use of the DTFE may lead to substantial improvements. A rather straightforward application is the reconstruction of the cosmic density field in the nearby universe. In Chapter 7 we have reconstructed the density field corresponding to the 2dF galaxy redshift survey. The resulting two- and three-dimensional maps reveal an impressive view on the cosmic structures in the nearby universe.

9.1.3 Numerical simulations of structure formation

In Chapter 5 we have described how the DTFE can be incorporated in particle hydrodynamics codes. We have shown that the improved density estimates of the DTFE will yield a major improvement for simulations incorporating feedback processes, which play a major role in galaxy and star formation. In this chapter we have also shown that the DTFE estimate has convenient properties that make the implementation of viscous forces better defined. The presented results form an encouraging step towards the insertion of the DTFE in astrophysical particle hydrodynamics codes.

9.1.4 The cosmic velocity field

The DTFE has been designed for reconstructing density or intensity fields from a discrete set of irregularly distributed points sampling this field. In Chapter 6 we have shown that it can also be used to reconstruct other continuous fields which have been sampled at the locations of these points. The use of the DTFE for this purpose has the same advantages as it has for reconstructing density fields. The fields are reconstructed locally without the application of an artificial or user-dependent smoothing procedure, resulting in an optimal resolution and the suppression of shot-noise effects. The estimated quantities are volume-covering and allow for a direct comparison with theoretical predictions. In this chapter we have focused on the simultaneous reconstruction of the density and velocity fields corresponding to cosmological

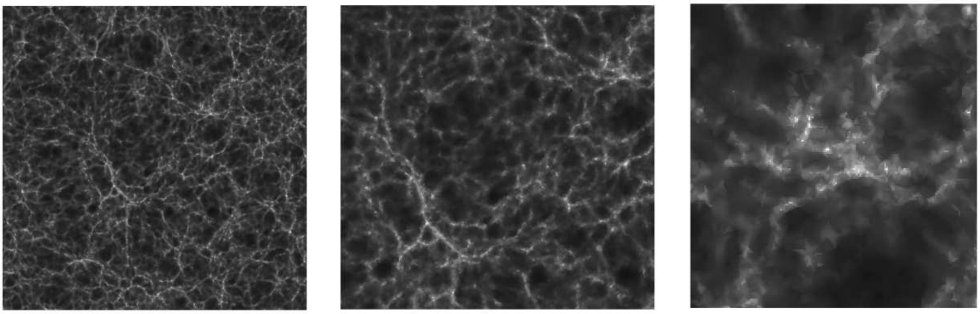

Figure 9.2 — Hierarchical clustering. Depicted is a slice through a cosmological N-body simulation (left-hand frame), together with a number of zoom-ins which focus on a low (central frame) and a high density (right-hand frame) region. On all depicted scales structures are present.

N-body simulations. The resulting fields closely adhere to the continuity equation. The DTFE reconstruction results in realistic density and velocity profiles without having to resolve to artificial smoothing procedures. The results show that the DTFE represents a major step forward for the analysis of cosmic velocity fields at both small and large scales.

9.1.5 Evolution and dynamics of the cosmic web

The DTFE has been specifically designed for describing the complex properties of the cosmic web. In Chapter 6 we have analyzed a number of simulations of cosmic structure formation. By means of a simultaneous DTFE reconstruction of the cosmic density and velocity field we have analyzed the dynamics of characteristic elements of the large scale galaxy distribution. We have described a number of analytic models of voids and shown that the DTFE reconstructions closely adhere to these models. We have also discussed how voids can be used to constrain the value of the cosmological constant. Finally, we have shown that the DTFE reconstructed density and velocity field near superclusters reproduces the theoretically expected infall patterns.

9.2 Virtues and limitations

In this section we shortly discuss the most important virtues and limitations of the DTFE with respect to conventional reconstruction procedures.

9.2.1 Virtues

9.2.1.1 Virtue: resolving hierarchical substructure

The first and foremost virtue of the DTFE is its self-adaptive nature. It automatically adapts to the local density of the sampling point distribution without depending on pre-fixed smoothing kernels or on other user-defined procedures. The key to the adaptiveness of the DTFE is the Delaunay tessellation of the sampling point distribution, which forms the heart of the DTFE reconstruction procedure. By definition the Delaunay tessellation is fully adaptive to the sampling point distribution. This property allows the DTFE to simultaneously resolve both high and low density regions. This is illustrated Fig. 9.2, in which an image of the DTFE

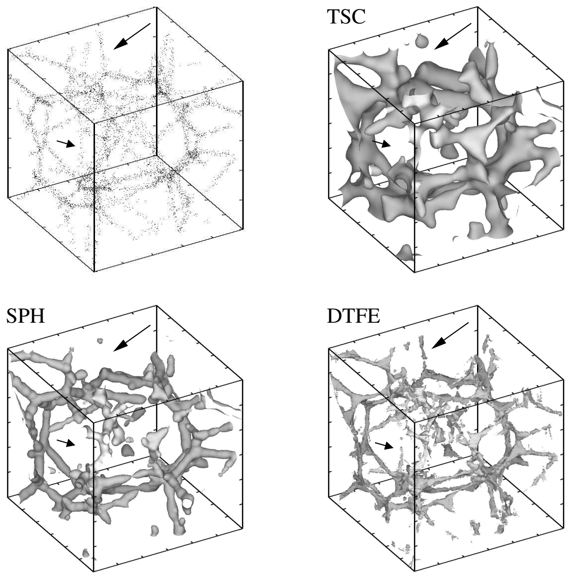

Figure 9.3 — Voronoi model of filamentary structure and the corresponding TSC, SPH and DTFE density field reconstructions. The density contours have been chosen such that 65% of the mass is enclosed. The arrows indicate two structures which are visible in both the galaxy distribution and the DTFE reconstruction, but which the conventional TSC and SPH reconstruction procedures do not resolve.

density field reconstruction of a slice through a cosmological N-body simulation is shown (left-hand frame). Clearly visible is that the slice contains all kinds of structures on different scales and of different density. The subsequent frames zoom in on a low and a high density region. In the central frame, where a low density void is depicted, one may observe that small-scale substructure is present within the void, while it is surrounded by flattened filamentary structures of relatively high densities. In the right-hand frame a high density clump of matter is depicted, which contains even smaller clumps of even higher density. The figure shows that the DTFE is able of simultaneously resolving both the large scale structure as well as the substructures contained within smaller structures.

9.2.1.2 Virtue: resolving anisotropy

The next important virtue of the DTFE is that it does not only adapt to the local density of the sampling point distribution, but also to its local geometry. In particular, the DTFE is able of resolving highly anisotropic structures, whereas conventional reconstruction schemes tend to smear such structures over a large volume, effectively making them more spherical. This is illustrated in Fig. 9.3, in which a three-dimensional Voronoi model of a filamentary network is shown, together with the corresponding TSC, SPH and DTFE density field reconstructions. The TSC reconstruction appears very different from the galaxy distribution. Individual structures do not resemble their counterparts in the galaxy distribution, but obtain a significantly larger volume. The resulting filamentary network is substantially more roundish and less

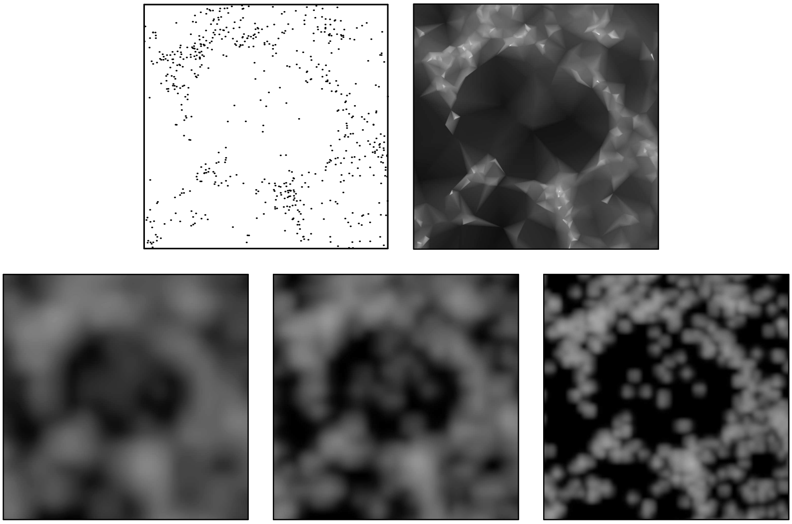

Figure 9.4 — Several density field reconstructions of a void region. Top left: galaxy distribution. Top right: DTFE reconstruction. Bottom row: TSC reconstructions at different resolution.

tenuous than the galaxy distribution. To a lesser extent the same is true for the SPH reconstruction. The DTFE reconstruction appears to resemble the galaxy distribution best. Note for example that at several locations structures are visible in the galaxy distribution which are also present in the DTFE reconstruction, but not in the TSC and SPH reconstructions (two examples are indicated by the arrows in Fig. 9.3).

9.2.1.3 Virtue: resolving low density regions

A third virtue of the DTFE is its capability of an accurate description of low density regions without introducing the shot-noise effects by which conventional methods are beset. This is illustrated in Fig. 9.4, in which the DTFE reconstruction of a void-like region is compared with several fixed grid-based TSC reconstructions at different resolutions. First look at the TSC reconstructions. It is clear that none of the adopted resolutions accurately describes the void. At the lowest resolution the overall shape of the void is recovered, but the sharp edges in the galaxy distribution are smeared into featureless blobs. To a lesser extent the same is true for the reconstruction at average resolution. Here, however, one starts to recognize the inner structure of the void breaking up into distinct density blobs, indicating that the resolution of the grid in the interior of the void is too high. This is very clearly visible in the high resolution reconstruction, in which the edges of the void seem to be described much better, but in which the interior of the void is dominated by shot-noise effects. Compare this with the DTFE reconstruction in the top right-hand frame. Here the sharp edges of the void are recovered as such, while at the same time the interior of the void is reconstructed as a gently varying low density region.

Although not shown in Fig. 9.4 SPH-like procedures represent a significant improvement over grid-based methods for low density regions. In Chapter 5 we have shown that the outer rims of filamentary and wall-like structures in SPH reconstructions are smeared into their neighbouring voids, overestimating the local density. DTFE reconstructed fields are not affected by this problem.

9.2.1.4 Virtue: reconstructing other dynamical fields

Apart from reconstructing density or intensity fields the DTFE may also and if desired simultaneously be used to reconstruct other continuous fields. The use of the DTFE for this purpose has the same advantages as for the reconstruction of density fields. This is illustrated in Fig. 9.5, in which a typical void-like region is shown together with the DTFE density and velocity field reconstructions. The thick line running from the bottom to the top of these fields indicates the one-dimensional section along which the density and velocity field are plotted in the bottom right-hand frame of the figure. The DTFE procedure clearly manages to render the void as a realistically slowly varying region of low density. Notice the clear distinction between the empty (dark) interior regions of the void and its edges. The velocity field shows that the void is expanding. This expansion is rather uniform as can be observed in the one-dimensional sections through the density and velocity reconstruction shown in the bottom right-hand frame of Fig. 9.5. The linear 'super-Hubble' expansion of voids is well understood in terms of gravitational dynamics. According to Birkhoff's theorem voids can be approximated as expanding, isolated universes unto themselves that do not accrete matter from the universe at large (e.g. van de Weygaert & van Kampen 1993, Goldberg & Vogeley 2004). Because voids are emptier than the rest of the universe they expand faster than the rest of the universe.

9.2.1.5 Virtue: dimensional independence

A final virtue we wish to mention is the fact that the DTFE is independent of the dimension of space. In this thesis we have applied it to both two- and three-dimensional fields. Recently, Arad et al. (2004, 2005) generalized the DTFE for computing the six-dimensional phase-space density $f(\mathbf{x}, \mathbf{v})$ and its PDF in an N-body system. They showed that f is a sensitive tool for studying the evolution of subhaloes during the hierarchical build-up of haloes.

9.2.2 Limitations

9.2.2.1 Limitation: sensitivity to sampling noise

The DTFE method has a self-adaptive spatial resolution and an effective smoothing kernel which is more localized than that of other reconstruction methods. It is therefore less forgiving with respect to sampling noise present in the data. Such noise will have a direct impact on the reconstructed field. In principle one may get rid of this noise by filtering the reconstructed density field in the post-processing stage. However, filtering destroys the adaptive capabilities of the DTFE which may be undesirable for many applications. It is therefore essential to understand how reconstructed structures are affected by sampling noise and how one may determine the significance of a reconstructed density field. This issue has been discussed in detail in Chapter 8.

9.2.2.2 Limitation: boundary conditions

An important practical issue for the DTFE is that of the boundary conditions. The problem resides in calculating density estimates at the border of the data point sample. If we

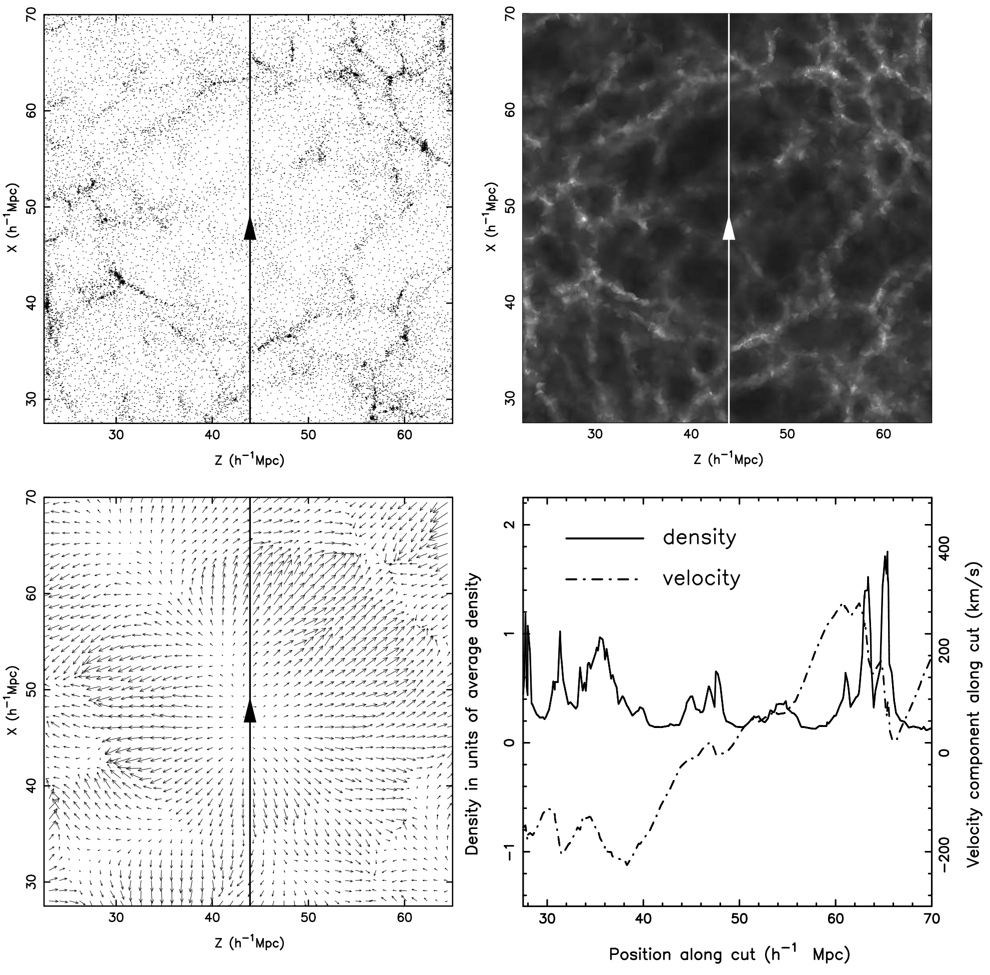

Figure 9.5 — Combined density and velocity field reconstructions. Top left-hand frame: particle distribution in a thin slice through the simulation box. Top right-hand frame: slice through the three-dimensional DTFE density field reconstruction. Bottom left-hand frame: slice through the three-dimensional DTFE velocity field reconstruction. Bottom right-hand frame: density and velocity reconstructions along the one-dimensional section indicated by the solid line shown in the other frames.

confine the Delaunay tessellation only within 'natural' edges imposed by the data set, the reconstructed density field will not be correct around the edges. This is because the outermost Delaunay cells stretch out into infinity. The volume of the outermost Delaunay cells is therefore infinitely large and the corresponding estimated densities at the location of the outermost sampling points are equal to zero.

Possible solutions to overcome this edge problem are: boundary padding, imposing periodic boundary conditions or using constrained realizations. Boundary padding refers to adding points beyond the edge of the sampling point distribution. The practical use mainly depends on the knowledge one has on the properties of the density field beyond the sampled region. For instance, for an object placed in a homogeneous background radiation field, the observed field may be extended by adding randomly distributed points.

A special case of boundary padding involves periodic boundary conditions, in which case the sampling point distribution is repeated beyond its edges. The practical use depends on the application. It is well suited for the analysis of cosmological N-body simulations, which usually involve periodic boundary conditions. The use of periodic boundary conditions for the analysis of real data is however rather limited.

A more advanced and consistent case of boundary padding would be to make use of the existing correlations in the field. Constrained field realizations (Bertschinger 1987, Hoffman & Ribak 1991, van de Weygaert & Bertschinger 1996) offer a natural solution for this strategy.

9.2.2.3 Limitation: linear artefacts

A third limitation of the DTFE is its linear nature. The density field is reconstructed as linearly varying with a gradient which is discontinuous at the edges of the Delaunay tetrahedra. At the scale of the local smoothing kernel, which is as large as the local contiguous Voronoi cell, this results in triangular artefacts, which form the imprint of the linear interpolation procedure. Physical density fields vary smoothly and are continuously differentiable. In principle it is possible to generalize the first-order DTFE reconstruction procedure to higher orders such as natural neighbor interpolation.

9.2.2.4 Limitation: empty regions

The final limitation of the DTFE we address here is that it is not capable of reconstructing regions of zero density. The reason for this is that any region with a finite size will correspond to Delaunay tetrahedra with a finite size and therefore a non-zero density.

9.3 Conclusions and outlook

In this thesis we have developed a new method for analyzing the large scale galaxy distribution. We have shown that this new method, the DTFE, performs significantly better than existing techniques. The applications we have described show that it can contribute to a variety of subjects in present-day cosmological research.

Several other astronomers have recognized the advantages of the DTFE and used it in their research. Bradac et al. (2004) used DTFE reconstructed surface density maps to compute the gravitational lensing pattern around galaxies, upon which Li et al. (2006) evaluated the method in its ability to trace higher-order singularities. Shandarin et al. (2004) advocate the DTFE for systematic studies of the size, shape and topology of the cosmic web by means of Minkowski functionals. Neyrinck et al. (2005) have used the DTFE to identify haloes in cosmological N-body simulations. Arad, Dekel & Klypin (2004) used the DTFE to assess the six-dimensional phase-space density distribution of dark haloes in cosmological N-body simulations (see also Arad & Johansson 2005).

In Groningen the cosmology group has applied the DTFE to several aspects of the large scale galaxy distribution. Romano-Díaz (2004) has constructed velocity maps of the nearby universe using the DTFE. Using the DTFE he has also characterized the thermal state of the Local Group and Local Supercluster by measuring volume-weighted cosmic Mach number statistics. He was able to show that the coldness of the local flow is due to the specific mass configuration beyond the Local Group. Besides studies concerning the cosmic velocity field the DTFE has also been used as the basis of two advanced algorithms for detecting characteristic structures in galaxy redshift surveys and cosmological N-body simulations. One of these techniques, the *Multiscale Morphology Filter* (Aragón-Calvo et al. 2006), is able of identifying clusters, filaments and walls, and is presently used for a systematic study of the properties of these structures. The other algorithm, the *Cosmic Watershed Algorithm*, is able of identifying voids (Platen & van de Weygaert 2006). Sophisticated applications like these, tuned towards uncovering the characteristics of the reconstructed large scale galaxy density field, yield the real potential of the DTFE.

We conclude that the DTFE forms a major step forwards for studies of the large scale galaxy distribution. This field of research is still advancing rapidly from forthcoming new data and bigger and more advanced numerical simulations. With better data and new theoretical and computational tools, such as the DTFE, cosmologists have the prospect of answering some of the fundamental open questions related to our cosmic world view.

References

Arad I., Dekel A., Klypin A., 2004, MNRAS, 353, 15

Arad I., Johansson P.H., 2005, MNRAS, 362, 252

Aragón-Calvo M., van de Weygaert R., Jones B.J.T., van der Hulst J.M., 2006, astro-ph/0610249

Bernardeau F., van de Weygaert R., 1996, MNRAS, 279, 693

Bertschinger E., 1987, ApJ, 323, 103

Bradac M. et al., 2004, A&A, 423, 13

Delone B.N., 1934, Bull. Acad. Sci. USSR: Classe Sci. Mat., 7, 793

Goldberg D.M., Vogeley M.S., 2004, ApJ, 605, 1

Hoffman Y., Ribak E., 1991, ApJ, 380, 5

Li G.-L. et al., 2006, astroph/0603557

Neyrinck M.C., Hamilton A.J.S., Gnedin N.Y., 2005, MNRAS, 362, 337

Okabe A., Boots B., Sugihara K., Nok Chiu S., 2000, *Spatial Tessellations, Concepts and Applications of Voronoi Diagrams*, 2nd edition, Wiley, Chichester, UK

Platen E., van de Weygaert R., 2006, *private communication*

Shandarin S.F., Sheth J.V., Sahni V., 2004, MNRAS, 353, 162

Sibson R., 1981, in *Interpreting Multi-Variate Data*, Barnet V. (ed.), Wiley, Chichester, UK

Sukumar N., 1998, PhD Thesis, Northwestern University, Evanston, IL, USA

Watson D.F., 1992, *Contouring: a Guide to the Analysis and Display of Spatial Data*, Pergamon Press, Oxford, UK

van de Weygaert R., Bertschinger E., 1996, MNRAS, 281, 84

van de Weygaert R., van Kampen E., 1993, MNRAS, 263, 481