

# Introduction

Let  $k$  be a finite field and  $k[t]$  a polynomial ring in one variable  $t$ . The ring  $k[t]$  shares many a property with the ring of integers  $\mathbf{Z}$ . This resemblance has been the subject of a long history of research, and up to today insights in one of the two arithmetical universes— $\mathbf{Z}$  and  $k[t]$ —continue to inspire the study of the other.

This thesis intends to make a contribution towards understanding the differences and similarities between ANDERSON'S  $t$ -motifs ( $k[t]$ ) and GROTHENDIECK'S motifs ( $\mathbf{Z}$ ). Such a comparison is bound to be rather formal, since  $t$ -motifs are defined in a direct way, while motifs arise from algebraic varieties in an implicit construction. The former are defined to be certain modules over a certain ring, the latter are objects from the target category of a universal cohomology theory of algebraic varieties. It should also be stressed that the existence of the latter (with its attributed properties) is only *conjectural*.

In Chapters 1, 2 and 3 of this manuscript a Tannakian formalism of  $t$ -motifs is set up. This means that a certain category of  $t$ -motifs is built that is equivalent with the category of representations of some affine group scheme  $\Gamma$ . Such a category is known, by the work of ANDERSON and PAPANIKOLAS. Our construction is similar, but differs in two points. The first is the lack of any finiteness conditions with respect to the skew polynomial ring  $K[\sigma]$ . This disconnects the theory from its more geometrical dual view-point of endomorphisms of additive group schemes, but also allows for greater flexibility. The second deviation is the use of the formal inversion of the Carlitz  $t$ -motif (the brother of the Lefschetz motif) in closing the theory under internal hom. This is of course in-

spired by the analogous construction for classical motifs. The resulting category is an explicit one in which the field of definition of an object and the isomorphism classes within an isogeny type become tangible.

In Chapter 4 it is shown that the group of components of  $\Gamma$  can be identified with the absolute Galois group of the base, modulo a certain fact that is proven in Chapter 8.

Chapter 5 studies a particular kind of  $t$ -motifs which, when compared with the classical motifs, could be considered pathological. Its abelian objects are studied and this leads to a universal pro-torus that is—perhaps surprisingly—very similar in nature to the pro-torus that plays a central role in SERRE’s study of abelian  $\ell$ -adic Galois representations.

Chapter 6 recalls the ‘ $t$ ’-theory of Diedonné modules (due to LAUMON) and relates it to weights of  $t$ -motifs. Nothing in this chapter is original, except possibly the philosophical digression which ends it.

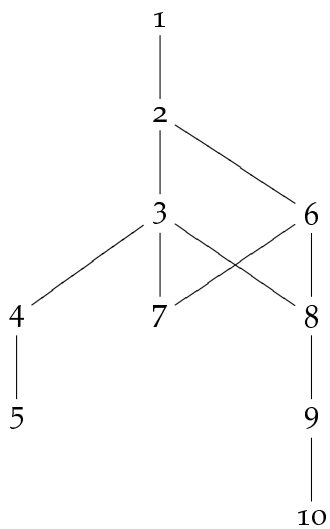
The category of  $t$ -motifs is not semi-simple. This is of course no surprise, given that the category of representations of an affine group scheme in characteristic  $p$  is almost never semi-simple. In Chapter 7 extensions of  $t$ -motifs are analysed, thus shedding some light on the obstructions to semi-simplicity.

Periods of  $t$ -motifs are introduced in Chapter 8. They can be interpreted as defining an ‘enriched’ fibre functor related to the infinite prime of  $k[t]$ . We prove that this functor is fully faithful on pure  $t$ -motifs. All this is very close to ANDERSON’s theory of scattering matrices. However our proof is direct, and does not fall back to geometrical considerations of additive group schemes. We should also mention that PINK has sketched an approach to constructing a period functor that can handle non-pure  $t$ -motifs properly. Unfortunately, no progress has been made on this front.

Finally, Chapters 9 and 10 are devoted to the study of some ‘Shimura varieties’ classifying  $t$ -motifs. More precisely, they use periods to give an analytic description of the moduli spaces introduced by LAUMON, RAPOPORT, and STUHLER in their proof of the local Langlands conjecture in characteristic  $p$ . These chapters stand a bit apart from the preceding

ones. Although at some points results from those earlier chapters are used, those could have been avoided—one can do with no more than the 1986 paper by ANDERSON.

The interdependence of the Chapters is summarised in the following diagram:





# Notation

Before we start off, some remarks on notation should be made.

Equalities, natural isomorphisms and fixed chosen identifications are all denoted by the sign '='. The addition of 'def' on top of the equality sign as in ' $\stackrel{\text{def}}{=}$ ' means that the left hand side is being *defined* to be equal to the right hand side. The symbol ' $\approx$ ' is used to indicate non-canonical isomorphism.

Let  $R$  be a ring and  $\tau : R \rightarrow R$  an endomorphism of  $R$ . An additive function  $\sigma$  between two left  $R$ -modules is said to be semi-linear with respect to  $\tau$  if it satisfies the identity  $\sigma(rm) = \tau(r)\sigma(m)$ . In this thesis (almost) all semi-linear functions are denoted by the Greek letter  $\sigma$ , and the different endomorphisms according to which they are semi-linear are all denoted by  $\tau$ . This will hopefully not lead to confusion.

Finally, we have adopted the not-so-very-standard notation  $A \rightsquigarrow B$  to indicate that  $B$  depends on  $A$  in a functorial way, in other words, that  $A \mapsto B$  is a functor.

