Chapter 11

Questions

In a 1985 interview, SERRE stated:

> ... Alas, most people are afraid to admit that they don't know the answer to some question, and as a consequence they refrain from mentioning the question, even if it is a very natural one. What a pity! As for myself, I enjoy saying "I do not know." [1]

With this wisdom in mind, we will end this thesis posing a number of questions that seem to be worth persuing further.

11.1. *If $k[t] \to K$ is injective, is the category $t\mathcal{M}^\circ(K)$ neutral Tannakian? And what about $t\mathcal{M}^\circ_{\text{int}}$?*

A related question is if every M has a Picard-Vessiot field with field of constants $k(t)$.

11.2. *Is there a direct proof of the analytic triviality of Drinfeld modules?*

The only proof I know is geometrical: it deduces the analytic triviality from DRINFELD's uniformisation, using the results of [ANDERSON 1986]. For the Carlits motif C there is a direct proof (3.2.3) and it seems likely that the something similar is possible for Drinfeld modules.

(1) *See* [CHONG AND LEONG 1985] or Œ. 141 in J.P. SERRE, *Collected Works*.

11.3. *The $k[t]$-modules* $\mathrm{Ext}(\mathbf{1}, C^i)$ *are invariants of the base field K. Do they have a conceptual interpretation?*

In the (conjectural) category of mixed motives, the groups of extensions of **1** by the powers of the Lefschetz motif are expected to coincide (at least up to torsion) with the *K*-groups of the base field.[(2)]

Probably more interesting are the subgroups of these Ext groups that consist of those extension classes that have a model over the 'ring of integers' of *K*.

11.4. *Does the vanishing of the Yoneda* Ext^i *for* $i > 1$ *in* $t\mathcal{M}^{\circ}_{\text{a.t.}}$ *imply any restrictions on* Γ*?*

One should be careful not to jump to conclusions on the cohomology of Γ: the vanishing of the higher Yoneda Ext groups in the category of finite-dimensional Γ-representations (7.4.2) does *not* imply that the higher Ext groups vanish in the larger category of all Γ-modules (in which the resolutions live that are needed to define cohomology of Γ.) It is therefore not possible to conclude that the higher cohomology groups

$$H^i(\Gamma, M) = \mathrm{Ext}^i_{\Gamma\text{-mod}}(\mathbf{1}, M)$$

vanish. (The reasoning in the proof of 7.4.2 can not be reversed.)

11.5. *What is the most general Shimura variety in the style of Chapter 10?*

The work of Voskuil and van der Put on rigid analytic symmetric spaces suggests that only products of forms of $\mathrm{GL}(n)$ yield algebraic quotients of symmetric spaces[(3)], but there are a few groups on which it is not conclusive.

(2) *See* for example §3 of [Deligne 1994].
(3) *See* [van der Put and Voskuil 1992].